You can actually solve the problem and find the transmission probabilities as a fn. of the angle of incidence – Katsnelson, Geim & Novoselov. (Nature Phys 2, 620 (2006).

They had found (for high barriers ($|V| >> |E|$))

$$T = \frac{\cos^2\varphi}{1 - \cos^2(q_x D)\sin^2\varphi}$$

as a fn. of the incident angle ϕ wrt x-axis and D = width of the well.

Finally, just to make connection with some more recent materials, let me mention topological insulators, Weyl semi-metals, silicene, etc. just to give you a flavour of some of the kinds of materials that are being studied and the ideas being used. All of these basically are also called Dirac materials because ~~they use~~ the dispersion of the electrons in these materials is relativistic — they use the Dirac eq^{n} and not the Schrodinger eq^{n}.

Topological insulators, unlike graphene which has 2 Dirac cones for each spin — so 4 Dirac cones in all, one for each inequivalent Fermi point and one for each spin, has only 1 Dirac cone. So it is often called 1/4 graphene.

on ... mension) T-

its surfaces in the bulk an

described by 2+1 (which are dimensional theories) —

the relevant ~~&~~ excitations are

described by a simple Dirac cone.

There is a lot of work been done

in this field in recent times

...t to tell you a little

more about it — let us first

... the symmetries in graphene.

...e are 2 important symmetries

... is parity which switches

... 2 sub-lattices, which is

...valent to rotating

$(\sigma_1, \sigma_2, \sigma_3) \to (\sigma_1, -\sigma_2, -\sigma_3)$

... the parity operator is just

$P = \sigma_1$

...ime-reversal operator T takes

$(\sigma_1, \sigma_2, \sigma_3) \to (\sigma_1, -\sigma_2, \sigma_3)$

$T = \sigma_0 \cdot K$

... conjugation, where K denotes

(This is $\|^r$ to what you may remember from 3+1 dimensions $P = \gamma_0$ & $T = \cancel{i\gamma_2\gamma_0}$ $\gamma^1\gamma^3 K$)

Here, the lattice fermions have a pseudo-spin (representing the sub-lattice isospin), but in terms of the real spin of the electron are spinless. That is why, the time-reversal operator obeys $T^2 = 1$. for real spin ½ electrons, $T^2 = -1$, which is what leads to Kramers degeneracy and the ~~phy~~ ~~stability~~ ~~o~~ physics associated with that.

For graphene, the main point is that the Hamiltonian given by

$$H = i\hbar v_0 \begin{pmatrix} 0 & k_x + ik_y \\ -k_x + ik_y & 0 \end{pmatrix}$$

~~xa~~ ~~to~~ & more generally written as

$$H = d_1(k)\sigma_1 + d_2(k)\sigma_2$$

is symmetric under P & Tr

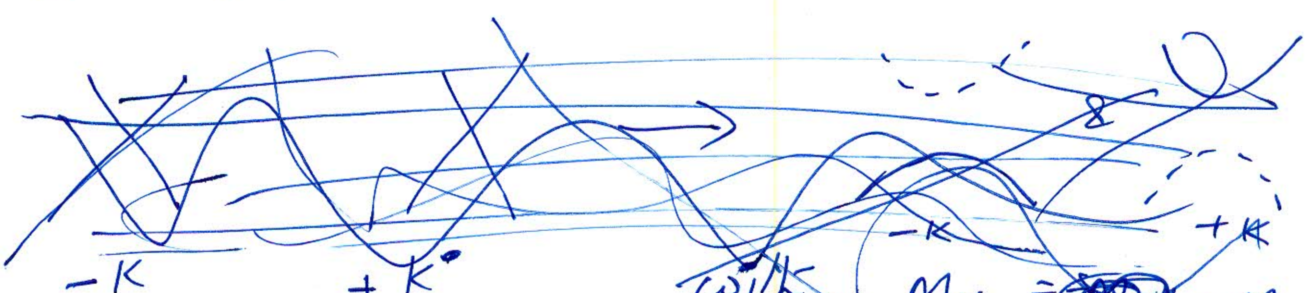
~~because~~

But if you add a term proportional to σ_3, then the Hamiltonian is no longer symmetric under P & T and one can generate gaps at the Dirac point.

One way to generate a mass is to just add a constant proportional to σ_3. ——, $M\sigma_3$.

This is called Semenoff mass and it corresponds to a staggered mass ~~—~~ — something which breaks the symmetry between the A & B sub-lattices, while leaving time-reversal symmetry intact.

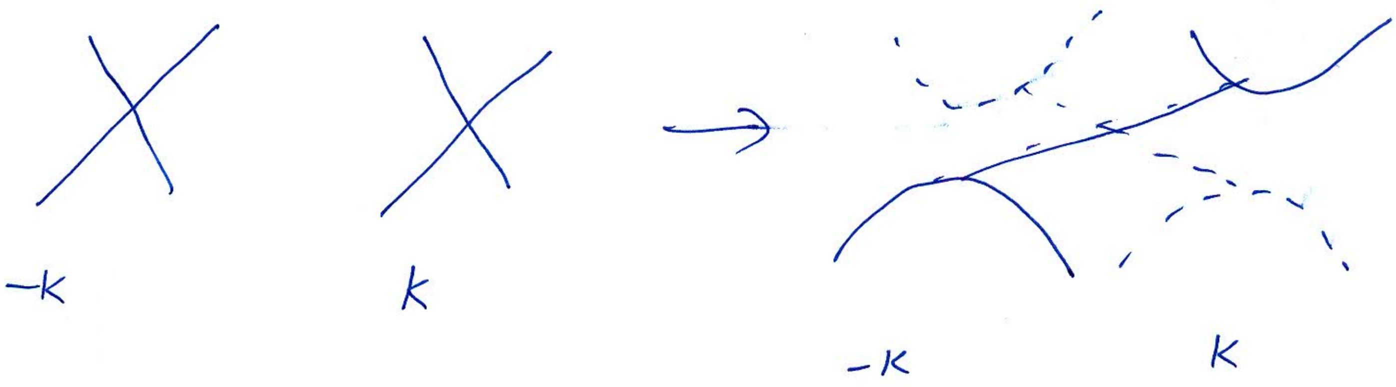
So ~~the~~ we will have $d_3(k) = M$



with $M_K = -M_{-K}$

This is realised in some crystals where the A & B sub-lattices are occupied by distinct atoms.

Then we have the Haldane mass. This breaks time-reversal symmetry, but not parity. In a lattice model, it can be generated by second nearest neighbours. But for us, the main point is that $d_3(k)$ changes sign ~~at the 2 valleys~~ and is not independent of k $\quad d_3(k \sim \pm K) = \mp A$



What is more interesting, if we ~~put then~~ solve this in a finite system (put graphene next to ~~an~~ an ordinary insulator). Then one will get edge states.

Such materials are called Chern insulators or quantum anomalous Hall insulators

Finally, if we also include real Spin, we get to the current excitement — topological insulators.

Here you have the same physics as in the Haldane phase, but ~~[illegible]~~ with the 2 edge states also having 2 distinct spins ↑ and ↓.

I don't want to go more into this field, but let me end this part of the course by mentioning a new ~~material~~ kind of material that we are currently working on. It is called Weyl semi-metal or also 3D graphene. ~~It is essentially the study of the Hamilton~~

It is found that in some ~~cases~~ materials / the easiest models are to start with topological insulators models and break TR or P invariance in such models — the actual that have ~~this~~ this kind of Hamiltonian as a low energy limit have very complicated band structures to begin with), the effective low energy Hamiltonian just turns out to be of the form

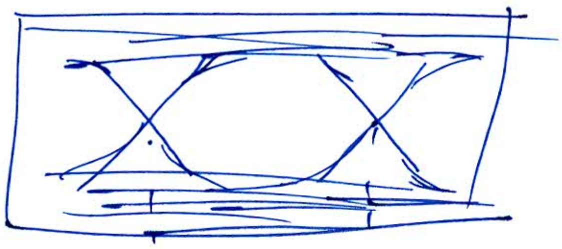
$$H = v \vec{p} \cdot \vec{\sigma}$$

— i.e. it is the Weyl Hamiltonian (for one helicity) in 3+1 dimensions. $H = -v\vec{p} \cdot \vec{\sigma}$ for the other helicity.

But here, the interesting point is that once the 2 helicities have been separated, it is not possible to add a mass term to the Hamiltonian (no 4th anti-commuting matrix — the reason we ~~sta~~ need 4 dim. matrices for the massive Dirac eqn in 3+1 dimensions). Hence the

Weyl Hamiltonean is not an insulator. It is metallic. It is called a semi-metal. Essentially, it is metallic only at the Dirac point – (low energy approximation) elsewhere it is insulating.

– e.g. the band structure of a WSM looks like



At particular k-points, the valence & conduction bands touch forming the WSM.

~~Finally~~

Finally, since I had mentioned silicene in my course outline. So, let me also mention what new materials like silicene and germanene are all about.

Silicene has Silicon atoms arranged in a honeycomb pattern, But with a buckled structure Just like in graphene, the states near the Fermi energy can be described by Dirac theory at the 2 valleys K and K'. Only difference is that the fermions are massive in silicon due to a spin-orbit coupling which is much larger in silicene than graphene. Also, because it is buckled. the 2 sub-lattices respond differently to an applied electric field. So this induces a gap, which is tunable. So added to the spin orbit gap. The total Dirac mass of ~~the~~ silicene can be tuned by an electric field. And so we can tune it to be a normal insulator, a metal like graphene, or a topological insulator by inverting the bands at K'. This makes it a very interesting material. People have also made something called germanene — sister to silicene and graphene. It seems to be flat, more like graphene, according to a recent 2014 paper but there hasn't been much work on this yet.

<u>References for this part</u>

1.) Introduction to Dirac materials and topological insulators — J. Cayssol. 1310.0792

2.) Relativistic quantum mechanics - vol I — Bjorken & Drell.